

The correct formulas for the number of partitions of a given number as a combination and as a permutation a missed discovery by Srinivasa Ramanujan due to non investigation of properties of basic underline arithmetic series

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Abstract

The newly discovered correct formulas are introduced for number partitions of a given number as a combination and as a permutation

Introduction

Finding the mathematical expression for partitions of a given number was a problem in mathematics attempted by Euler, Jacobi, Cauchy, Major Mac Mahon, Srinivasa Ramanujan, J. Tannery, J. Molk, G. H. Hardy, George Andrews and many others. Ramanujan¹⁻⁵ suggested an asymptotic formula for large n , the number of partitions of a given number and with assistance of Major Mac Mahon he was attempting to come up with digits of the partitions for $n=100$. But it was an excessive estimate which is thousand times bigger than the correct value that we are going to calculate by very simple but most elegant arithmetic series beautifully entering into mathematics. But his first three digits are correct. If Ramanujan studied basic arithmetic series very carefully at pre university college standard of Kumbakonam presidency college he could not have missed the opportunity of finding this very simple formulas. Ramanujan has wasted his time and life going to England an unhealthy journey. He could have saved his life and time being in Madras a healthy city for long survival in India. He was advised sea voyage is not good. But he went to England via Colombo and Bombay & got sickness upon there and in his return to Madras via Bombay. The discovery that we are going to report was made on twenty fourth of April in the year 2015 two days before the day Ramanujan has died on twenty sixth of April in 1920 ninety five years before and further results on third of May in year 2015 the Vesak full moon day. I have been observing the documentary film on Srinivasa Ramanujan by Google internet and in his lecture on Ramanujan's work in mathematics George Andrews was pointing out the number of partitions of 7 is fifteen. I slept for few minutes and got up suddenly and observed the rest of the film and many mathematicians were highlighting Ramanujan's mathematics and enlightening the film. Following morning after deep sleep when I had a look at the Robert Kanigel's Book on Ramanujan the man who knew Infinity and in his book he was pleasing the exact mathematical expressions from readers I started to write the number of partitions of a given number starting from one. Every problem that I have address in Mathematics vanished with an astonishing solution. This time it happened with the number of partitions of a given number that Ramanujan had predicted the existence of such an expression but it was very difficult for him to find. This is what I have found.

Let $P_c(n)$ be the number of combinations of the partitions of n independent of the order and Let $P_p(n)$ be the number of permutations of the partitions of n with order.

$$P_c(1)=P_c(2)=1, P_c(3)=2, P_c(4)=4, P_c(5)=6, P_c(6)=10, P_c(7)=15, \dots$$

$$P_p(1)=P_p(2)=1, P_p(3)=3, P_p(4)=7, P_p(5)=15, P_p(6)=27, P_p(7)=43, \dots$$

$$P_c(2)-P_c(1)=0=1-1$$

$$P_c(3)-P_c(2)=1=2-1$$

$$P_c(4)-P_c(3)=2=3-1$$

$$P_c(5)-P_c(4)=2=4-1-1$$

$$P_c(6)-P_c(5)=4=5-1$$

$$P_c(7)-P_c(6)=5=6-1$$

$$\begin{array}{ccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

$$P_c(n-1)-P_c(n-2)=n-3$$

$$P_c(n)-P_c(n-1)=n-2$$

$$\underline{P_c(n+1)-P_c(n)=n-1}$$

$$P_c(n+1)-P_c(1)=n-1+n-2+n-3+\dots\dots\dots+4+3-1+2+1+0=\{(n-1)n/2\}-1$$

$$P_c(n+1)=(n-1)n/2$$

$$\underline{P_c(n)=(n-2)(n-1)/2}$$

$$P_c(100)=(100-2)(100-1)/2=98 \times 99/2=49 \times 99=4851$$

$$P_p(2)-P_p(1)=0$$

$$P_p(3)-P_p(2)=2$$

$$P_p(4)-P_p(3)=4=1 \times 4=(3-2) \times 4$$

$$P_p(5)-P_p(4)=8=2 \times 4=(4-2) \times 4$$

$$P_p(6)-P_p(5)=12=3 \times 4=(5-2) \times 4$$

$$P_p(7)-P_p(6)=16=4 \times 4=(6-2) \times 4$$

$$\begin{array}{ccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

$$\underline{P_p(n+1)-P_p(n)=(n-2) \times 4}$$

$$P_p(n+1)-P_p(1)=0+2+\{1+2+3+4+\dots\dots+(n-2)\} \times 4$$

$$P_p(n+1)=3+\{(n-2)(n-1)/2\} \times 4=3+2(n-2)(n-1)$$

$$P_p(n+1)=3+P_c(n) \times 4$$

$$\underline{P_p(n)=3+P_c(n-1) \times 4}$$

$$\underline{P_p(n)=3+2(n-3)(n-2)}$$

$$P_p(100)=3+P_c(99) \times 4=3+\{97 \times 98/2\} \times 4=3+2 \times 98 \times 97=\mathbf{19015}$$

Ramanujan's Asymptotic Formula for number of partitions of n for large n is given by

$$P(n) \approx e^{\pi \sqrt{2n/3}} / 4n \sqrt{3}$$

$$P(100) \approx e^{3.1415 \sqrt{(200/3)}} / 400 \sqrt{3}$$

$$P(100) \approx 190569291.996 \text{ Major MacMohan-Ramanujan value}$$

$$100^2=10,000$$

190569291.996/10,000=19056.9291996 has first three digits correct but other estimates of them do not produce at least such digits. So Ramanujan had obtained ten thousand times bigger values for number of partitions of n for 100. So that Srinivasa Ramanujan from Kumbakonam of Tanjore district in Madras that is Chennai of Modern India came up with first three digits correctly but having very large uncertainties in comparison with our most reliable estimates. So S. Ramanujan the Indian Giant of Mathematics had really missed basic understanding of number pattern of partitions of a given number and underlying arithmetic behind & it was a school mathematical sequence if he had concentrated more on arithmetic series and on the difference in between combinations and permutations which are school mathematics lessons had he properly referred some other basic mathematics texts. He did not pass university entrance exam three times and had missed basic college education at Kumbakonam presidency college and lack of experience and bad luck. Namagiri the Goddess of Namakkal could have helped him with simple arithmetic sequence patterns in his night dreams than Euler patterns that really misdirected his mathematical thinking the thought process if he made attention on psychology he could have get admission to college under

competition. He could have avoided writing to G.H. Hardy and settle in India to have long life. Why he worried going to British Isle? Even India had good Mathematicians at the time under British education. Even School education could have been sufficient for S. Ramanujan. What Ramanujan thought asymptotically he can come up but still correct general mathematical functions should exist.

For writing corresponding generating functions for partitions of given n

we introduce $P_c(0) = P_p(0) = 1$ for $x^0 = 1$

$f_c(x) = 1 + x + x^2 + 2x^3 + 4x^4 + (1/2)\sum(n-1)(n-2)x^n$ for combinations

$f_p(x) = 1 + x + x^2 + \sum\{3+2(n-2)(n-3)\}x^n$ for permutations

mathematical formulas expected by Euler and Ramanujan but could never be found which beautifully enter into mathematics. It is very difficult to say Euler or Ramanujan can provide these coefficients out of their products in their denominators by expanding and collecting x to power terms. Due to difficulties Ramanujan had to face we removed all hurdles in Mathematics. Now Ramanujan got cured from mathematical sickness originated from Euler patterns a fashion. Mathematics is now recovered.

In hospital doctors suspected Ramanujan had cancer, tuberculosis, malnutrition since he was a strict vegetarian an Indian Tamil Brahmin. He went to England in year 1914 by Steam Ship Nevasa and returned back to Madras in 1920 by Steam Ship Nagoya via Colombo Harbour. As a honour to his journey via Lanka one jetty of Colombo harbour should be named as Srinivasa Ramanujan Jetty and one ship should also be named as Srinivasa Ramanujan Ship. Carr's Synopsis of elementary results in pure mathematics contains Spherical Trigonometry having known them Ramanujan must have assisted the ship captain for its journey on earth sphere the path up to England and return, from Chennai of modern India and he was accounts clerk of the port trust of Madras. In his sea journey and return he was severely effected by wind. Upon return to Madras he got bone, skin & muscle pain. Let us consider the series

$$1/(1-x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + \dots + x^{n-2} + x^{n-1} + x^n + \dots$$

By differentiating it with respect to x we get

$$1/(1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 + 9x^8 + 10x^9 + 11x^{10} + 12x^{11} + \dots + (n-2)x^{n-3} + (n-1)x^{n-2} + nx^{n-1} + \dots$$

By differentiating it with respect to x again

$$2/(1-x)^3 = 2 + 3.2x + 4.3x^2 + 5.4x^3 + 6.5x^4 + 7.6x^5 + 8.7x^6 + 9.8x^7 + 10.9x^8 + 11.10x^9 + 12.11x^{10} + \dots + (n-2)(n-3)x^{n-4} + (n-1)(n-2)x^{n-3} + n(n-1)x^{n-2} + \dots$$

$$1/(1-x)^3 = 1 + 3x + 2.3x^2 + 5.2x^3 + 3.5x^4 + 7.3x^5 + 4.7x^6 + 9.4x^7 + 5.9x^8 + 11.5x^9 + 6.11x^{10} + \dots + \{(n-2)(n-3)/2\}x^{n-4} + \{(n-1)(n-2)/2\}x^{n-3} + \{n(n-1)/2\}x^{n-2} + \dots$$

$$= 1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + 28x^6 + 36x^7 + 45x^8 + 55x^9 + 66x^{10} + \dots + \{(n-2)(n-3)/2\}x^{n-4} + \{(n-1)(n-2)/2\}x^{n-3} + \{n(n-1)/2\}x^{n-2} + \dots$$

$$1 + x + x^2 + x^3 + x^4 + \{x^3/(1-x)^3\} = 1 + x + x^2 + 2x^3 + 4x^4 + 6x^5 + 10x^6 + 15x^7 + 21x^8 + 28x^9 + 36x^{10} + 45x^{11} + 55x^{12} + 66x^{13} + \dots + \{(n-2)(n-3)/2\}x^{n-1} + \{(n-1)(n-2)/2\}x^n + \{n(n-1)/2\}x^{n+1} + \dots$$

for combinations

$$-2 - 2x - 2x^2 + \{3/(1-x) + 4x^4/(1-x)^3\} = 1 + x + x^2 + 3x^3 + 7x^4 + 15x^5 + 27x^6 +$$

$$43x^7 + 63x^8 + 85x^9 + 115x^{10} + 145x^{11} + 183x^{12} + 223x^{13} + 267x^{14} + \dots + \{3+2(n-2)(n-3)\}x^n +$$

$$\{3+2(n-1)(n-2)\}x^{n+1} + \{3+2n(n-1)\}x^{n+2} + \dots$$

for permutations

That is the long awaited final solution but it was hard to achieve from the time of Leonard Euler. Now Ramanujan must feel comfortable and Hardy should be happy. It is coming not from mathematical patterns of Euler that was a mere fashion. If Ramanujan made little more attention on geometric

series for x less than one and worked on pre university basic school calculus specially differentiation he could have come up but he really missed the discovery unfortunately due to bad luck. These are not very difficult but one has to think rightly. If you go on wrong track you will not achieve the target.

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \sinh x + \cosh x = e^x,$$

$$p(n) = \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}} = \frac{\sinh \pi\sqrt{2n/3} + \cosh \pi\sqrt{2n/3}}{4n\sqrt{3}} \text{ for large } n$$

$$p(100) = \frac{\sinh \pi\sqrt{200/3} + \cosh \pi\sqrt{200/3}}{400\sqrt{3}} \text{ for } n=100$$

$$= \frac{2\sqrt{3} \sinh(10\pi\sqrt{6}/3)}{1200} = \frac{\sqrt{3} \sinh(3.33\pi\sqrt{6})}{600} = \frac{\sqrt{3} \cosh(3.33\pi\sqrt{6})}{600} \text{ for large } x, \text{ since } e^{-x} \rightarrow 0$$

$$= \frac{1.7320 \sinh(25.650)}{600} = \frac{1.7320 \cosh(25.650)}{600} = \underline{199275047.691130}$$

If you divide it by $100^2 = 10,000$ we get 19927.5047691130 only first two digits are correct

independent of the order of magnitude which is not at all correct, this is in attempt to help

Ramanujan in his former achievements with Mac Mahon asymptotically as conjectured by Euler mean while 19015 is the value of $p(100)$ estimated by us.

Song

Sinhx plus Coshx exponential x

Ramanujan's asymptotic formula in hyperbolic x

Scientific calculator of a computer

Do not provide answers to exponentials

Ramanujan not at all up to the order of magnitude

But nearer to first three digits in the case of one hundred

Ramanujan left to England via Colombo harbour

Better if he could have become a barber

He reached to England by S.S. Nevasa

Returned to Madras by S.S. Nagoya

What a big booster Ramanujan was

Euler and Hardy are the other two

Euler misguided his mathematical thought

Formula asymptotical that Ramanujan sought

Euler wrote seventy volumes

Ramanujan came up with five volume

Hardy published six volumes

Just one more

None of them ever solved any problem

Confused the mind with mental illusions

Nonsense

Ramanujan was at ship bar

Did he study spherical trigonometry by Carr

Could he assist the captain of the ship

On his way to Cambridge

Ship travel on earth sphere

Mathematics gives Captain fear

Logarithms, Trigonometry on a sphere

Are the discoveries of Napier

Wind whistled Ramanujan's ear

Ramanujan was a strict vegetarian

What a meal could Captain offer him

Tamil Brahmin by faith
Mathematics was his taste

Film

Euler,Ramanujan and Hardy are very big boosters in Mathematics.

Film name is **Three Big Boosters**

Conclusions

We have completely solved the problem on partitions of a given number,the number of ways of writing a number as sums of positive integers which are whole numbers and arrived at correct formulas for combinations and permutations .Now all congruence relations of Ramanujan have non zero residuals.Even only with first three terms of the infinite product in the denominator of Euler's conjecture that is by restricting/lifting/relaxing the unrestriction imposed , the problem cannot be solved so it must be an incorrect conjecture.Euler's collection of coefficients for different powers of x gives different results not the correct number of partitions of a given number as the coefficient of x to power n . Even for very low values of n ,number of partitions given by Euler looks incorrect.

Acknowledgement

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